FULLY WORKED SOLUTIONS

Chapter 18: Making music

Chapter questions

1. (a) $L = 0.6$ m, $v = 400$ m s^{-1}

   $\lambda_1 = 2L = 1.2$ m

   $f_1 = v/\lambda_1 = 400/1.2 = 333$ Hz

   (b) $L = 0.3$ m, $v = 400$ m s^{-1}

   $\lambda = 2L = 0.6$ m

   $f = v/\lambda = 400/0.6 = 667$ Hz

   (c) $L = 0.6$ m

   $\lambda_2 = L = 0.6$ m

   $f_2 = v/\lambda_2 = 400/0.6 = 667$ Hz

2. They are equal.

3. $\lambda_A = 2\ L_A = 2 \times 0.9 = 1.8$ m

   $f_A = 440$ Hz

   $v = f_A\lambda_A = 440 \times 1.8 = 792$ m s^{-1}

   $\lambda_C = v/f_A = 792/512 = 1.55$ m

   $L_C = \lambda_C/2 = 1.55/2 = 0.78$ m

4. $f_3 = 3f_1 = 3 \times 250 = 750$ Hz

5. $\lambda_1 = 4L = 4 \times 1.40 = 5.6$ m

   $f_1 = v/\lambda_1 = 344/5.6 = 61.4$ Hz

6. $\lambda_2 = 4L/3 = 4 \times 2.75/3 = 3.7$ m

7. $\lambda = v/f = 344/256 = 1.34$ m

   $L = \lambda/4 = 1.34/4 = 0.336$ m $= 33.6$ cm

Review questions

5.

String	No. of nodes	No of antinodes	λ	Harmonic
B	2	1	$2L$	1st
D	5	4	$L/2$	4th
C	3	2	L	2nd
A	4	3	$2L/3$	3rd

6. $f_4 = 880$ Hz

(a) $f_n = nf_1$

$$f_1 = \frac{f_4}{n} = \frac{880}{4} = 220 \text{ Hz}$$

(b) $f_3 = 3f_1 = 3 \times 220 = 660$ Hz

7. $L = 0.8$ m, $v = 240$ m s^{-1}

(a) $\lambda = 2L = 1.6$ m

(b) $f = \frac{v}{\lambda} = \frac{240}{1.6} = 150$ Hz

(c) 0.2 m

(d) $\lambda = \frac{3L}{2} = \frac{3 \times 0.8}{2} = 1.2$ m

8. (a) $\lambda = 0.4$ m, $n = 3$

$$\lambda = \frac{nL}{2}$$

$$0.4 = \frac{3L}{2}$$

$L = 0.27$ m

(b) $n = 2$

$$\lambda = \frac{nL}{2}$$

$$0.4 = \frac{2L}{2}$$

$$L = 0.4 \text{ m}$$

(c) $v = f\lambda = 400 \times 0.4 = 160$ m s^{-1}

9. (a)

$$L = \frac{n\lambda}{2} \text{ therefore } \lambda_n = \frac{2L}{n}$$

$$\lambda_1 = \frac{2 \times 0.6}{1} = 1.2 \text{ m}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{340}{1.2} = 283 \text{ Hz}$$

$$\lambda_2 = \frac{2 \times 0.6}{2} = 0.6 \text{ m}$$

$$f_2 = \frac{v}{\lambda_2} = \frac{340}{0.6} = 567 \text{ Hz}$$

$$\lambda_3 = \frac{2 \times 0.6}{3} = 0.4 \text{ m}$$

$$f_3 = \frac{v}{\lambda_3} = \frac{340}{0.4} = 850 \text{ Hz}$$

(b)

$$\lambda_n = \frac{4L}{2n - 1}$$

$$\lambda_1 = \frac{4 \times 1.2}{2 \times 1 - 1} = 4.8 \text{ m}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{340}{4.8} = 70.8 \text{ Hz}$$

$$\lambda_2 = \frac{4 \times 1.2}{2 \times 2 - 1} = 1.6 \text{ m}$$

$$f_2 = \frac{v}{\lambda_2} = \frac{340}{1.6} = 212.5 \text{ Hz}$$

$$\lambda_3 = \frac{4 \times 1.2}{2 \times 3 - 1} = 1.2 \text{ m}$$

$$f_3 = \frac{v}{\lambda_3} = \frac{340}{1.2} = 345 \text{ Hz}$$

(c)

$$\lambda_n = \frac{4L}{2n - 1}$$

$$\lambda_1 = \frac{4 \times 0.3}{2 \times 1 - 1} = 1.2 \text{ m}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{340}{1.2} = 283 \text{ Hz}$$

$$\lambda_2 = \frac{4 \times 0.3}{2 \times 2 - 1} = 0.4 \text{ m}$$

$$f_2 = \frac{v}{\lambda_2} = \frac{340}{0.4} = 850 \text{ Hz}$$

$$\lambda_3 = \frac{4 \times 0.3}{2 \times 3 - 1} = 0.24 \text{ m}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{340}{0.24} = 1420 \text{ Hz}$$

(d)

$$\lambda_n = \frac{2L}{n}$$

$$\lambda_1 = \frac{2 \times 2}{1} = 4 \text{ m}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{340}{4} = 85 \text{ Hz}$$

$$\lambda_2 = \frac{2 \times 2}{2} = 2 \text{ m}$$

$$f_2 = \frac{v}{\lambda_2} = \frac{340}{2} = 170 \text{ Hz}$$

$$\lambda_3 = \frac{2 \times 2}{3} = 1.3 \text{ m}$$

$$f_3 = \frac{v}{\lambda_3} = \frac{340}{1.3} = 255 \text{ Hz}$$

(e)

$$\lambda = \frac{4L}{2n - 1}$$

$$\lambda_1 = \frac{4 \times 2}{2 \times 1 - 1} = 8 \text{ m}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{340}{8} = 42.5 \text{ Hz}$$

$$\lambda_2 = \frac{4 \times 2}{2 \times 2 - 1} = 2.33 \text{ m}$$

$$f_2 = \frac{v}{\lambda_2} = \frac{340}{2.33} = 146 \text{ Hz}$$

$$\lambda_3 = \frac{4 \times 2}{2 \times 3 - 1} = 1.60 \text{ m}$$

$$f_3 = \frac{v}{\lambda_3} = \frac{340}{1.6} = 212.5 \text{ Hz}$$

10. 524 Hz = 131 $\times$ 2^2. Therefore, 524 Hz is 2 octaves above 131 Hz.

11. In air, v = 344 m s^{-1}, f = 200 Hz

$$\lambda = \frac{v}{f} = \frac{344}{200} = 1.72 \text{ m}$$

As $\lambda_1 = \frac{4L}{2 \times 1 - 1} = 4L$,

$$L = \frac{1.72}{4} = 0.43 \text{ m}$$

That is, the pipe must be 43 cm long.

When submerged in water, where v = 1498 m s^{-1}:

$$f = \frac{v}{\lambda} = \frac{1498}{1.72} = 871 \text{ Hz}$$

12. (b) For a closed pipe, there will be half a wavelength between the resonant lengths. Therefore, if there is 33 cm between resonant lengths, the wavelength must be 66 m.

As f = 524 Hz,

$v = f\lambda = 524 \times 0.66 = 346$ m

13. With all of the finger holes covered, the flute will act as an open pipe at its first resonant length, so $\lambda = 2L$.

$$\lambda = \frac{v}{f} = \frac{344}{262} = 1.31 \text{ m}$$

$$L = \frac{\lambda}{2} = \frac{1.31}{2} = 0.66 \text{ m}$$

14. $L = 0.70$ m, $f = 330$ Hz

$\lambda = 2L = 1.4$ m

$v = f\lambda = 330 \times 1.4 = 462 \text{ m s}^{-1}$ in the string

When $f = 440$ Hz,

$$\lambda = \frac{v}{f} = \frac{462}{440} = 1.05 \text{ m}$$

$$L = \frac{\lambda}{2} = \frac{1.05}{2} = 0.525 \text{ m}$$

As the string is 0.70 m long, the string will need to be stopped a distance (0.7 – 0.525) = 0.175 m from the top of the string.

15. (a) As it is a closed pipe, $\lambda = 4L = 4 \times 2.5 = 10$ cm = 0.1 m

$$f = \frac{340}{0.1} = 3400 \text{ Hz}$$

(b) This is within the range of hearing. (Refer to the diagram on page 428.)

16. They vary in wire composition, tension and diameter (gauge). As a result, sound travels at a different speed in each of the strings. This way, they vibrate at different frequencies even though the lengths of the strings are the same.

17. Let D be the distance between the workers. D = 600 m

The time taken for sound to travel this distance in air from the impact point is:

$$t_{\text{air}} = \frac{D}{v_{\text{air}}} = \frac{600}{344} = 1.74 \text{ s}$$

The time taken for sound to travel this distance in steel from the impact point is:

$$t_{\text{steel}} = \frac{D}{v_{\text{steel}}} = \frac{600}{5400} = 0.11 \text{ s}$$

Therefore, the time difference will be equal to (1.74 – 0.11) = 1.63 s

19. $d = 1.6$ cm = 0.016 m

$L_{\text{effective}} = L_{\text{actual}} + 0.58d$

$L_{\text{effective}} = L_{\text{actual}} + 0.58 \times 0.016$

$L_{\text{effective}} = L_{\text{actual}} + 0.00928$

$$\%\ \text{error} = \frac{L_{\text{effective}} - L_{\text{actual}}}{L_{\text{actual}}} \times 100\%$$

$$0.05 = \frac{0.00928}{L_{\text{actual}}}$$

$$L_{\text{actual}} = \frac{0.00928}{0.05} = 0.186\ \text{m}$$

The pipe would need to be 18.6 cm long to cause a 5 % between the actual and the effective lengths.

21. $\alpha_{\text{steel}} = 11 \times 10^{-6}\ °\text{C}^{-1}$

$\Delta L = \alpha L \Delta T$

Let L be the original length of the pipe in Australia.

$L = 2$, $n = 1$

$$\lambda = \frac{4L}{2n - 1} = 4L$$

$$\lambda = 4 \times 2 = 8\ \text{m}$$

$\Delta L = 2 \times 11 \times 10^{-6} \times (-66) = -1.45 \times 10^{-3}$ m

Length in Russia = 2 m – 0.0014 m = 1.9986 m

$\lambda = 4L = 4 \times 1.9986 = 7.994$ m

$$v_{\text{Australia}} = 331\sqrt{\frac{T}{273} + 1}$$

$$v_{\text{Australia}} = 331\sqrt{\frac{26}{273} + 1} = 346.4\ \text{m s}^{-1}$$

$$f_{\text{Australia}} = \frac{v}{\lambda} = \frac{346.4}{8} = 43.3\ \text{Hz}$$

$$v_{\text{Moscow}} = 331\sqrt{\frac{-40}{273} + 1} = 305.8\ \text{m s}^{-1}$$

$$f_{\text{Moscow}} = \frac{v}{\lambda} = \frac{305.8}{11.9986} = 25.5\ \text{Hz}$$

$\Delta f = 43.3 - 25.5 = 17.8$ Hz

The difference in pitch would be difficult to pick for most people. Also, keep in mind:

- The other organ pipes would also change in length proportionately, so the instrument would still stay in tune with itself.

- The organ would no doubt be housed in a concert hall or cathedral, where the temperature would be nowhere near as low.